UNIVERSITÀ DI PISA

ALGEBRA 2

ANDREA BANDINI

Anno accademico	2021/22
CdS	MATHEMATICS
Codice	038AA
CFU	6

Moduli	Settore/i	Tipo	Ore	Docente/i
ALGEBRA 2	MAT/02	LEZIONI	60	ANDREA BANDINI ENRICO SBARRA

Learning outcomes

Knowledge
At the end of the course students should acquire knowledge of basic structures of commutative algebra and of their properties and should be able to apply such knowledge to various area of mathematics (e.g. algebraic geometry). Moreover students should be able to understand and elaborate statements and proofs concerning the specific topics of the course. In particular, students must have acquired knowledge about the tools and methodologies concerning: ideals, modules, exact sequences, tensor products, localization, primary decompositions, Artinian and Noethian rings and modules, Gröbner bases and computations on polynomial rings.

Assessment criteria of knowledge
Methods:

Final oral exam
Final written exam
Periodic written tests
Exercise sheets avaliable on the teachers' websites

Skills
The student must be able to understand and elaborate the proofs of the theorems discussed during the course and deduce other properties which follow from these theorems. Moreover, he/she must be able to solve in a rigorous manner exercises regarding the topics covered in class, applying methods and theorems presented during the course.

Assessment criteria of skills
Exercise on the topics discussed during the lectures are avaliable on various textbooks and on the teachers' websites, with those and through discussions with teachers and colleagues, the student will be able to check his/her level of understanding.

Behaviors
Students will be able to discuss the topics of the course in a rigorous manner and to solve non-trivial exercise and problems related to such topics. In particular students will acquire a geometric viewpoint on the subject in preparation for a future course on algebraic geometry, and a greater taste for constructive methods that can be implemented via algorithms in preparation for a future course in computational algebra.

Assessment criteria of behaviors
Students will verify their understanding of the topics presented in the course and their ability to solve exercises by discussing with teachers and colleagues and by comparing their solutions with them.

Prerequisites
A very good knowledge of arithmetic and of the basic algebraic structures (groups, fields). In particular the student should have succesfully taken courses on Arithmetic and Algebra I.

Teaching methods
Delivery: face to face.

Learning activities:

- attending lectures
- individual study
- group work

Attendance: not mandatory but advised.

Syllabus

Commutative rings: commutative rings with unity, ideals, prime and maximal ideals, nilpotents, quotient rings. Homomorphisms and homomorphism theorems. Chinese remainder theorem. PID and UFD, local rings. Polynomial rings, spec of a ring, affine varieties, Hilbert Nullstellensatz.

Modules: modules and submodules, finitely generated modules, Nakayama's lemma. Homomorphisms of A-modules, homomorphism theorems, direct products and tensor products, projective modules, flat modules. Exact sequences, Hom functors. Free modules, torsion modules, finitely generated modules over PID.

Localization: multiplicative systems, ring of fractions, correspondence between ideals in A and in S^{-1}A. Functorial properties of S^{-1}, local properties, localization at one prime.

Noetherian rings: Noetherian and Artinian rings, Noetherian and Artinian modules. Primary ideals and irreducible ideals, primary decomposition in Noetherian rings.

Gröbner bases: monomial ideals, monomial orders, initial ideals, S-polynomials and Buchberger algorithm. Minimal and reduced Gröbner bases. Applications to ideals and relations with properties of affine algebraic varieties.

Bibliography

Lecture notes are avaliable on the teacher's website. The topics presented in the course are treated in several textbooks on Commutative Algebra. Recommended reading:

M. Artin "Algebra" (2nd edition)

M.F. Atiyah - I.G. Macdonald "Introduction to Commutative Algebra"

D. Cox - J. Little - D. O'Shea, "Ideals, Varieties and Algorithms"

D. Eisenbud "Commutative Algebra with a view toward Algebraic Geometry"

E. Kunz "Introduction to Commutative Algebra and Algebraic Geometry"

H. Matsumura "Commutative Ring Theory"

M.Reid "Undergraduate Commutative Algebra"

Non-attending students info

Please check all the information on the course website.

Assessment methods

The exam consists of:

- written exam
- oral exam

In the written exam students must show their knowledge writing effective and rigorous solutions for some exercises requiring the application of results presented inthe course. During the oral exam students must show their mastery of the course topics by correctly exposing definitions, theorems and proofs, thus proving their understanding of the course material.
There will be written tests during the course: one after (approximately) half the course and one at the end, before the first summer exam: passing the two tests allows the student to be admitted to the oral exam in any of the exams of the summer session.

Additional web pages

Andrea Bandini homepage:

https://sites.google.com/site/banand207/home

Enrico Sbarra homepage:

http://people.dm.unipi.it/sbarra/#

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