



# UNIVERSITÀ DI PISA

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## CONDENSED MATTER PHYSICS

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CdS FISICA  
Codice 370BB  
CFU 9

Moduli	Settore/i	Tipo	Ore	Docente/i
CONDENSED MATTER PHYSICS	FIS/03	LEZIONI	54	MARCO POLINI FABIO TADDEI

### Obiettivi di apprendimento

#### Conoscenze

Il corso tratta argomenti di Fisica Teorica rilevanti per lo studio di sistemi fisici d'interesse per la materia condensata. Esso consiste in un nocciolo di 36 ore di argomenti di base essenziali per chiunque voglia specializzarsi in Fisica Teorica della Materia: la teoria della risposta lineare, la teoria diagrammatica delle perturbazioni, e la teoria di Landau dei liquidi di Fermi normali. Nella seconda parte di 24 ore si tratteranno argomenti avanzati, spesso d'interesse contemporaneo. Nell'Anno Accademico 2022/2023 verranno discusse teorie elementari per lo studio del trasporto termoelettrico in sistemi a stato solido quali la teoria semiclassica di Boltzmann e la teoria quantistica di Landauer-Büttiker (12 ore). Si passerà quindi a trattare la fisica dell'effetto Hall quantistico in regime frazionario introducendo i concetti di carica elettrica frazionaria e spiegando l'origine dell'incompressibilità nel livello di Landau più basso (6 ore). Si concluderà discutendo un modello esattamente solubile (modello di Sachdev-Ye-Kitaev) come esempio paradigmatico di sistema a molti corpi che non è possibile inquadrare nell'ambito della teoria di Landau dei liquidi di Fermi normali (6 ore).

#### Modalità di verifica delle conoscenze

Prova orale su progetto individuale.

#### Prerequisiti (conoscenze iniziali)

Si consiglia di aver seguito il corso di Fisica Teorica 1 o qualsiasi corso ove venga spiegata la seconda quantizzazione.

#### Programma (contenuti dell'insegnamento)

##### Linear response theory (12 hrs)

General theory of linear response: assumptions and derivation of the Kubo formula; the case of periodic perturbations; exact-eigenstate (Lehmann) representation; the fluctuation-dissipation theorem; analytic properties and Kramers-Kronig dispersion relations; the stiffness theorem; examples of linear-response functions: response to scalar and vector potentials: density-density versus longitudinal and transverse current-current response; gauge invariance; local and non-local optical conductivity; the f-sum rule; the orbital magnetic susceptibility; linear response of independent fermions: the particle-hole continuum; the static limit; the dynamical limit.

Case study: the interacting electron gas (Coulomb interactions); screened potential and dielectric function; proper versus full response function; the compressibility sum rule; the random phase approximation (RPA): pros and cons; plasmons (the roles of spatial dimensionality and Galilean invariance).

##### Suggested readings

Modern textbooks: Chapter 7: A. Altland and B. Simmons, *Condensed Matter Field Theory* (Second Edition) (Cambridge University Press, Cambridge, 2010); Chapters 3,4, and 5: G.F. Giuliani and G. Vignale, *Quantum Theory of the Electron Liquid* (Cambridge University Press, 2005).

Classics: Chapters 3 and 4: D. Pines and P. Nozières, *The Theory of Quantum Liquids* (W.A. Benjamin, Inc., New York, 1966);

##### Many-body diagrammatic perturbation theory (12 hrs)

Green's functions and field theory for fermions (zero temperature). Schrödinger picture, interaction picture, and Heisenberg picture; adiabatic "switching on"; Gell-Mann and Low theorem on the ground state in quantum field theory; Green's functions: definitions; relation to observables; example: free fermions; Lehmann representation; physical interpretation of the Green's function; Wick's theorem; diagrammatic analysis of perturbation theory: Feynman diagrams in coordinate space; Feynman diagrams in momentum space; Dyson equation; irreducible self-energy; the spectral function; the Hartree-Fock approximation; effective interactions; the Bethe-Salpeter equation.

Field theory at finite temperature for fermions. Definition of temperature Green's functions; relation to observables; example: free fermions; perturbation theory and Wick's theorem at finite temperature; diagrammatic analysis of perturbation theory: Feynman diagrams in coordinate space; Feynman diagrams in momentum space; Matsubara sums.

##### Suggested readings

Classics: Chapters 3,4,7 and 8: A.L. Fetter and J.D. Walecka, *Quantum Theory of Many-Particle Systems* (Dover, 2003).

##### Landau theory of Fermi liquids (12 hrs)



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Macroscopic theory: the Landau free-energy functional; the Landau parameters; specific heat, compressibility, and spin susceptibility; the quasiparticle lifetime; microscopic underpinning of the Landau theory: the spectral function; quasiparticle self-energy, renormalization constant, and effective mass; Luttinger theorem; approximate theories of the quasiparticle self-energy ("GW"); Landau kinetic equation; response of quasiparticles to external perturbations; transition from the collisionless to the collisional (hydrodynamic) regime; Fermi liquids from a renormalization-group approach to interacting fermions.

### Suggested readings

Modern textbooks: Chapter 8: G.F. Giuliani and G. Vignale, "*Quantum Theory of the Electron Liquid*" (Cambridge University Press, 2005).

Classics: Chapters 1 and 3: D. Pines and P. Nozières, "*The Theory of Quantum Liquids*" (W.A. Benjamin, Inc., New York, 1966).

Research articles: R. Shankar, *Rev. Mod. Phys.* **66**, 129 (1994); I. Torre *et al.*, *Phys. Rev. B* **99**, 144307 (2019).

### Selected topics (24 hrs)

#### Semiclassical and quantum theories of electron transport (12 hrs)

The Boltzmann equation; electrical conductivity; mean free path and carrier mobility; electron-impurity scattering; "ideal" resistance, i.e. electron-phonon scattering; the Bloch-Grüneisen law; general transport coefficients; thermal conductivity: the Wiedemann-Frank law; thermo-electric effects. Theory of quantum transport: the Landauer-Büttiker scattering formalism; derivation of charge and heat currents; conductance quantization; ballistic and diffusive transport regimes (relation to Ohm's law); current fluctuations; derivation of the current in hybrid superconducting systems; Andreev scattering processes; multiple scattering formula.

### Suggested readings

Modern textbooks: Chapter 1: Y.V. Nazarov and Y.M. Blanter, "*Quantum Transport*" (Cambridge University Press, 2009);

Chapters 2 and 3: S. Datta, "*Electronic Transport in Mesoscopic Systems*" (Cambridge University Press, 1997).

Classics: Chapter 7: J.M. Ziman, "*Principles of the Theory of Solids*" (Cambridge University Press, 1972).

Research articles: Y.M. Blanter and M. Büttiker, *Phys. Rep.* **336**, 1 (2000); C.J. Lambert and R. Raimondi, *J. Phys.: Condens. Matter* **10**, 901 (1998).

#### The fractional quantum Hall effect (6 hrs)

Electrons in the lowest Landau level (LLL); Vandermonde polynomials; the Laughlin wave function for odd-denominator fractional quantum Hall states; the one-component classical plasma analogy; fractionally charged quasiparticles; Bijl-Feynman theory of liquid Helium; variational upper bound on the excitation energy; phonons and rotons in  $^4\text{He}$ ; the origin of incompressibility in the fractional quantum Hall regime; projection into the lowest Landau level (LLL); non-commutative quantum mechanics in the LLL; Girvin-MacDonald-Platzman magneto-roton theory of intra-Landau-level collective excitations; projected f-sum rule; projected structure factor of a Laughlin wave function; collective excitations at long-wavelengths: chiral gravitons.

### Suggested readings

Modern textbooks: Chapter 10: G.F. Giuliani and G. Vignale, "*Quantum Theory of the Electron Liquid*" (Cambridge University Press, 2005).

Further reading: R.B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983); S.M. Girvin and T. Jach, *Phys. Rev. B* **29**, 5617 (1984); S.M. Girvin, A.H. MacDonald, and P.M. Platzman, *Phys. Rev. B* **33**, 2481 (1986); K. Yang, *Phys. Rev. B* **93**, 161302(R) (2016); F.D.M. Haldane, *Phys. Rev. Lett.* **107**, 116801 (2011); S.-F. Liou, F.D.M. Haldane, K. Yang, and E.H. Rezayi, *Phys. Rev. Lett.* **123**, 146801 (2019).

#### Quantum matter without quasiparticles (6 hrs)

The aim of these lectures is to present and discuss the analytical solution of the Sachdev-Ye-Kitaev model in the limit of large  $N$ , based both on the use of Feynman diagrams and path integrals evaluated at the saddle point.

### Suggested readings

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **70**, 3339 (1993); A.Y. Kitaev, Talks at KITP (LINKS:

<https://online.kitp.ucsb.edu/online/entangled15/kitaev/> and <https://online.kitp.ucsb.edu/online/entangled15/kitaev2/>), University of California, Santa Barbara (USA), Entanglement in Strongly Correlated Quantum Matter (2015); Y. Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky, arXiv:1910.14099; V. Rosenhaus, *J. Phys. A: Math. Theor.* **52**, 323001 (2019).