



# UNIVERSITÀ DI PISA

## PARTIAL DIFFERENTIAL EQUATIONS

VLADIMIR SIMEONOV GUEORGUIEV

Anno accademico	2022/23
CdS	MATHEMATICS
Codice	545AA
CFU	6

Moduli	Settore/i	Tipo	Ore	Docente/i
EQUAZIONI ALLE DERIVATE PARZIALI	MAT/05	LEZIONI	48	VLADIMIR SIMEONOV GUEORGUIEV

### Learning outcomes

#### Knowledge

The student who successfully completes the course will be able to manipulate PDEs by using elementary (but efficient) techniques.

#### Assessment criteria of knowledge

Written and oral exam.

#### Skills

The student will be able to manipulate partial differential equation by using elementary techniques. In particular:  
characteristic methods for first order PDEs, boundary and Cauchy problems, maximum principles in several forms, convergence to the initial datum, interpolation theory, further properties on Lebesgue measure and classical functional spaces.  
Basic tools in real analysis as maximal functions, convergence almost everywhere, Lebesgue derivation theorem and a.e. convergence to initial datum for the heat flow, Hardy-Littlewood-Sobolev inequality.  
Uniqueness criteria for the linear heat equation and finite propagation speed for the wave equation.

#### Assessment criteria of skills

Written and oral exam.

#### Behaviors

To provide basic knowledge in partial differential equations by using elementary tools.

#### Assessment criteria of behaviors

Written and oral exam.

#### Prerequisites

Theory of multivariable functions, ordinary differential equations,  $L^p$  spaces and basic knowledge of Lebesgue measure.

#### Teaching methods

Delivery: face to face

#### Syllabus

Spaces  $C^k$ , Holder spaces. Sobolev spaces in  $\mathbb{R}^n$  via the Fourier transform. Recalls on the trace and inequalities of Sobolev, Young, Poincaré.  
Recalling harmonic functions.  
Poisson equation in  $\mathbb{R}^n$ . Green function for Dirichlet problem.  
Maximum principle. the case of elliptic operators. Applications: Schawder estimates.  
Helmholtz equation. Resolvent of Laplace operator in  $\mathbb{R}^n$  or in bounded domain with boundary.  
The idea of Peron method and the layer potential method.  
Heat equations. convergence to the initial data and introduction to the concept of maximal function. Uniqueness criteria of the solution and



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examples of non-uniqueness. Introduction to Navier - Stokes equations.

Wave equation and properties of the solutions (finite speed of propagation, behaviour for long times, etc.)

Other equations of mathematical physics: Maxwell, Schrodinger, Klein - Gordon, Dirac.

First-order equations and method of characteristics. Hamilton - Jacobi equations. Shock creation.

### [Bibliography](#)

J. Rauch, An introduction to PDEs

L. Evans, Partial differential equations

F. John, Partial differential equations

Notes provided by the teacher.

### [Non-attending students info](#)

To study suggested textbooks as well as the notes of the course provided by the teacher.

It will be also very important to solve exercises proposed on the textbook as well as during the lectures and on the notes of the course.

*Updated: 20/01/2023 12:26*