



# UNIVERSITÀ DI PISA

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## CONDENSED MATTER PHYSICS

MARCO POLINI

Anno accademico	2023/24
CdS	FISICA
Codice	370BB
CFU	9

Moduli	Settore/i	Tipo	Ore	Docente/i
ADVANCED CONDENSED MATTER PHYSICS	FIS/03	LEZIONI	54	GIACOMO MAZZA MARCO POLINI DAVIDE ROSSINI FABIO TADDEI ANDREA TOMADIN

### Obiettivi di apprendimento

#### Conoscenze

Il corso tratta argomenti di Fisica Teorica rilevanti per lo studio di sistemi fisici d'interesse per la materia condensata. Esso consiste in un nocciolo di 36 ore di argomenti di base essenziali per chiunque voglia specializzarsi in Fisica Teorica della Materia: la teoria della risposta lineare, la teoria diagrammatica delle perturbazioni, e la teoria di Landau dei liquidi di Fermi normali. La seconda parte del corso consta di un totale di 18 ore in cui si tratteranno argomenti in un certo senso "avanzati", spesso d'interesse contemporaneo. Nell'Anno Accademico 2023/2024 offriamo due alternative:

- La prima alternativa (18 ore) tratterà la superconduttività e la fisica di un modello esattamente solubile (modello di Sachdev-Ye-Kitaev), paradigma di sistemi a molti corpi del tipo "Non-Fermi Liquids".
- La seconda alternativa (sempre di 18 ore) si concentrerà sulla fisica dell'effetto Hall quantistico in regime frazionario (introducendo i concetti di carica elettrica frazionaria e spiegando l'origine dell'incompressibilità nel livello di Landau più basso) e sulle teorie elementari per lo studio del trasporto termoelettrico in sistemi a stato solido (quali la teoria semiclassica di Boltzmann e la teoria quantistica di Landauer-Büttiker).

#### Modalità di verifica delle conoscenze

Prova orale su progetto individuale.

#### Prerequisiti (conoscenze iniziali)

Si consiglia di aver seguito il corso di Fisica Teorica 1 o qualsiasi corso ove venga spiegata la seconda quantizzazione.

#### Programma (contenuti dell'insegnamento)

##### Core part of the course (36 hrs)

##### A) Introduction to the theory of electron liquids (8 hrs)

Jellium model and Coulomb interaction regularization; the electron density as coupling constant; real-space orbital-based representation: Slater determinants; field theoretic representation of the jellium model; ground state of the free Fermi gas; density-of-states as a function of energy; kinetic energy of the free Fermi gas; first-order perturbation theory in the Coulomb interaction: the exchange energy; orbital picture of Hartree-Fock theory: from atoms and molecules to the jellium model; density-density correlator: the pair correlation function; ground-state energy theorem and integration-over-the-coupling constant algorithm.

##### B) Linear response theory (12 hrs)

General theory of linear response: assumptions and derivation of the Kubo formula; the case of periodic perturbations; exact-eigenstate (Lehmann) representation; the fluctuation-dissipation theorem; analytic properties and Kramers-Kronig dispersion relations; the stiffness theorem; examples of linear-response functions: response to scalar and vector potentials: density-density versus longitudinal and transverse current-current response; gauge invariance; local and non-local optical conductivity; the f-sum rule; the orbital magnetic susceptibility; linear response of independent fermions: the particle-hole continuum; the static limit; the dynamical limit. Case study: the interacting electron gas (Coulomb interactions); screened potential and dielectric function; proper versus full response function; the compressibility sum rule; the random phase approximation (RPA): pros and cons; plasmons (the roles of spatial dimensionality and Galilean invariance).

##### Suggested readings

Modern textbooks: Chapter 7: A. Altland and B. Simmons, *Condensed Matter Field Theory* (Second Edition) (Cambridge University Press, Cambridge, 2010); Chapters 3,4, and 5: G.F. Giuliani and G. Vignale, *Quantum Theory of the Electron Liquid* (Cambridge University Press, 2005).

Classics: Chapters 3 and 4: D. Pines and P. Nozières, *The Theory of Quantum Liquids* (W.A. Benjamin, Inc., New York, 1966).

##### C) Many-body diagrammatic perturbation theory (8 hrs)



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Green's functions and field theory for fermions (zero temperature). Schrödinger picture, interaction picture, and Heisenberg picture; adiabatic "switching on"; Gell-Mann and Low theorem on the ground state in quantum field theory; Green's functions: definitions; relation to observables; example: free fermions; Lehmann representation; physical interpretation of the Green's function; Wick's theorem; Feynman diagrams in momentum space; Dyson equation; irreducible self-energy; the spectral function; the Hartree-Fock approximation.

### Suggested readings

Classics: Chapters 3,4,7 and 8: A.L. Fetter and J.D. Walecka, "*Quantum Theory of Many-Particle Systems*" (Dover, 2003).

### D) Landau theory of Fermi liquids (8 hrs)

Macroscopic theory: the Landau free-energy functional; the Landau parameters; specific heat, compressibility, and spin susceptibility; the quasiparticle lifetime; microscopic underpinning of the Landau theory: the spectral function; quasiparticle self-energy, renormalization constant, and effective mass; approximate theories of the quasiparticle self-energy ("GW").

### Suggested readings

Modern textbooks: Chapter 8: G.F. Giuliani and G. Vignale, "*Quantum Theory of the Electron Liquid*" (Cambridge University Press, 2005).

Classics: Chapters 1 and 3: D. Pines and P. Nozières, "*The Theory of Quantum Liquids*" (W.A. Benjamin, Inc., New York, 1966).

Research articles: R. Shankar, *Rev. Mod. Phys.* **66**, 129 (1994); I. Torre *et al.*, *Phys. Rev. B* **99**, 144307 (2019).

### First module (18 hrs)

#### A) Superconductivity (12 hrs)

Basic phenomenology of superconductors; Fermions with attractive interaction; BCS theory. Anomalous Green's functions. Unconventional superconductivity from repulsive interaction.

### Suggested readings

Tinkham. Introduction to superconductivity.

Abrikosov, Gorkov, Dzhalyoshinski. Methods of quantum field theory in statistical physics.

#### B) Quantum matter without quasiparticles (6 hrs)

The aim of these lectures is to present and discuss the analytical solution of the Sachdev-Ye-Kitaev model in the limit of large N, based both on the use of Feynman diagrams and path integrals evaluated at the saddle point.

### Suggested readings

S. Sachdev and J. Ye, *Phys. Rev. Lett.* **70**, 3339 (1993); A.Y. Kitaev, Talks at KITP (LINKS:

<https://online.kitp.ucsb.edu/online/entangled15/kitaev/> and <https://online.kitp.ucsb.edu/online/entangled15/kitaev2/>), University of California,

Santa Barbara (USA), Entanglement in Strongly Correlated Quantum Matter (2015); Y. Gu, A. Kitaev, S. Sachdev, and G. Tarnopolsky,

arXiv:1910.14099; V. Rosenhaus, *J. Phys. A: Math. Theor.* **52**, 323001 (2019).

### Second module (18 hrs)

#### A) Semiclassical and quantum theories of electron transport (12 hrs)

The Boltzmann equation; electrical conductivity; mean free path and carrier mobility; electron-impurity scattering; "ideal" resistance, i.e. electron-phonon scattering; the Bloch-Grüneisen law; general transport coefficients; thermal conductivity; the Wiedemann-Frank law; thermo-electric effects. Theory of quantum transport: the Landauer-Büttiker scattering formalism; derivation of charge and heat currents; conductance quantization; ballistic and diffusive transport regimes (relation to Ohm's law); current fluctuations; derivation of the current in hybrid superconducting systems; Andreev scattering processes; multiple scattering formula.

### Suggested readings

Modern textbooks: Chapter 1: Y.V. Nazarov and Y.M. Blanter, "*Quantum Transport*" (Cambridge University Press, 2009);

Chapters 2 and 3: S. Datta, "*Electronic Transport in Mesoscopic Systems*" (Cambridge University Press, 1997).

Classics: Chapter 7: J.M. Ziman, "*Principles of the Theory of Solids*" (Cambridge University Press, 1972).

Research articles: Y.M. Blanter and M. Büttiker, *Phys. Rep.* **336**, 1 (2000); C.J. Lambert and R. Raimondi, *J. Phys.: Condens. Matter* **10**, 901 (1998).

#### B) The fractional quantum Hall effect (6 hrs)

Electrons in the lowest Landau level (LLL); Vandermonde polynomials; the Laughlin wave function for odd-denominator fractional quantum Hall states; the one-component classical plasma analogy; fractionally charged quasiparticles; Bijl-Feynman theory of liquid Helium; variational upper bound on the excitation energy; phonons and rotons in 4He; the origin of incompressibility in the fractional quantum Hall regime; projection into the lowest Landau level (LLL); non-commutative quantum mechanics in the LLL; Girvin-MacDonald-Platzman magneto-roton theory of intra-Landau-level collective excitations; projected f-sum rule; projected structure factor of a Laughlin wave function; collective excitations at long-wavelengths: chiral gravitons.

### Suggested readings

Modern textbooks: Chapter 10: G.F. Giuliani and G. Vignale, "*Quantum Theory of the Electron Liquid*" (Cambridge University Press, 2005).

Further reading: R.B. Laughlin, *Phys. Rev. Lett.* **50**, 1395 (1983); S.M. Girvin and T. Jach, *Phys. Rev. B* **29**, 5617 (1984); S.M. Girvin, A.H.

MacDonald, and P.M. Platzman, *Phys. Rev. B* **33**, 2481 (1986); K. Yang, *Phys. Rev. B* **93**, 161302(R) (2016); F.D.M. Haldane, *Phys. Rev. Lett.*

**107**, 116801 (2011); S.-F. Liou, F.D.M. Haldane, K. Yang, and E.H. Rezayi, *Phys. Rev. Lett.* **123**, 146801 (2019).