UNIVERSITÀ DI PISA

MATHEMATICS FOR NEUROSCIENCES

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CdS	NEUROSCIENCE
Codice	623AA
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Moduli	Settore/i	Tipo	Ore	Docente/i
MATHEMATICS FOR NEUROSCIENCES	MAT/07	LEZIONI	56	RITA GIULIANO

Programma (contenuti dell'insegnamento)

Linear Algebra
Introduction to linear algebra. Definition of R^n. Properties of vectors in R^n. General definition of vector space. Definition of vector subspace. Linear combinations. Linearly independent vectors. Generators of a vector space. Definition of basis of a vector space. Canonical basis in R^n. Dimension of a vector space. Finite dimensional vector spaces.
Implicit and parametric equation of a straight line in R^2. Methods of transforming one form into the other one. Parametric equation of a straight line in R^3. Implicit and parametric equation of a plane in R^3 .
Linear maps: definitions and properties. Kernel and image of a linear map. Grassmann formula.
Matrices. Representation of linear maps through matrices (canonical basis for both the domain and the codomain). Product (rows by columns) of matrices. Composition of linear maps. Product matrix as a representation of composition of linear maps. Methods for determining the matrix associated to a linear map. Examples in R^2: simmetries with respect to some particular lines (through the origin).
The algebra of matrices: properties of the sum amd the product of matrices. Example for the NON commutativity of the product. Square matrices. Identity matrix. Inverse matrix. Necessary and sufficient condition for a matrix to be invertible (columns or rows must form a basis).
Determinant of a square matrix: interpretation in R^2 and in R^3. Proprerties of the determinant. Calculation of the determinant of a matrix 2x2 and 3x3 (Sarrus' rule). Calculation of the determinant via the Gauss-Jordan method. Necessary and sufficient condition of invertibility of matrices (through the determinant). Calculus of the inverse of a 2x2 matrix. Calculation of the inverse of a matrix via the Gauss-Jordan method.
Definition of eigenvalues and eigenvectors. Characteristic polynomial. Examples of calculations of eigenvalues and eigenvectors.. Diagonalization. Sufficient condition for diagonalization. Diagonalization of symmetric matrices.

Descriptive Statistics
Introduction to Statistics. Descriptive Statistics: samples of data, characters, modality of a character, numerical (discrete and continuous) characters, non numerical characters, absolute frequency, relative frequency, cumulative frequency, graphical representations of data.
Mode. Arithmetic mean: definition and properties. Definition of quantile, median. Dispersion measures: range, interquartile interval, variance and its properties.
Correlation of two characters: regression line, covariance, correlation coefficient and its properties. Chebicev inequality. Skewness e kurtosis.
Multivariate Analysis: covariance matrix. Principal components Analysis (PCA).

Probability Theory
Introduction to Probability Theory: sample space, events, rules for the calculation of probabilities of events. Deterministic and random phenomena. Mathematical models of random phenomena. The probability of an event as degree of confidence. Additivity of probability. Conditional probability. Independence of events. Bayes formula.
Some applications to genetics: Hardy-Weinberg law; the phenomenon of dominance.
The notion of random variable. Discrete random variables. Density of a discrete random variable. Main discrete densities: binomial, geometric, negative binomial, Poisson.
Independence of random variables. Bernoulli trials scheme.
The distribution function of a random variable.
The Poisson Process.
Absolutely continuous random variablese. Gaussian densities. The standard Gaussian density. Properties of the density and the distribution function of the standard Gaussiana law. Standardization of random variables. Use of the table of the N(0,1) distribution.
The notion of expectation (mean) of a random variable.. Examples: mean of the Binomial, geometric, negative binomial, Poisson distribution. Moments of a random variable. Variance. Properties of the variance. Covariance. Chebicev inequality. Calculation of the mean and the variance of a Gaussian random variable. The statement of the Central Limit Theorem. Normal approximation formula and its use. Calculation of the law of the sample mean of a Gaussian sample. The weak Law of Large Numbers: statement and proof via the Chebicev inequality.

Inferential Statistics
The chi squared distribution. The Student's t- distribution. Use of the tables of quantiles for the chi-squared and Student's t-distributions.
The statement of Cochran's Theorem.
The concept of estimator. Unbiased estimators. Unbiased estimators for the mean and the variance. Risk of an estimator..
Maximum likelihood estimators. Maximum likelihood equation. Examples.
The concept of quantile. Quantiles of the N(0,1), the chi-squared and the Student's t-distribution.
Definition of confidence interval. Construction of the most relevant confidence intervals for gaussian samples. The pivotal quantity method.
Introduction to the theory of tests. Critical region, level, power. Gaussian tests. Detailed description of the bilateral test for the mean of a gaussian sample.
The P-value and its use.
Description of the chi-squared test: Pearson statistics, critical region of the test. The case of estimated parameters. The case of an infinite-valued distribution. Description of the independence test.

Multivariable Calculus
Functions of more than one variable: some topological notions in R^n. The concept of limit, continuity, partial derivability, differenziability. The connection of the various concepts. The total differential Theorem. Applications of the total differential theorem. The notion of tangent plane to a surface. The notion of directional derivative.
Connection between the directional derivative and the gradient. Definition of Jacobian matrix. The chain rule for functions of more than one variable. Some particular cases.
The complex numbers: definition, cartesian and trigonometric form, operations with complex numbers, complex conjugate. Complex exponential.

Introduction to the theory of stochastic processes
Autocovariance and autocorrelation functions, coefficient of autocorrelation.
Weak (or wide-sense) and strong stationary processes. White noise. Gaussian processes. Time series in l^2 and in l^1. Convolution of time series. Definition of the discrete Fourier transform (DFT). Fourier inverse transform.
Properties of the discrete Fourier transform. Fourier transform of a truncated time series. Definition of power spectrum. Power spectrum of the White Noise. Colored noises. The statement of the Wiener-Khinchin Theorem.

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